MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

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2. Write your answers in this Question/Answer Booklet using a blue / black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1 **(7 marks)**

Differentiate with respect to x. Simplify your answers, leaving them in fully factorised form where appropriate.

(a) $\dfrac{2x+1}{8-3x}$ (2 marks)

(b) $(2x^4 + 2)(4 + x)^8$ (3 marks)

(c) $\sin(3x) - 4\cos 2x$ (2 marks)

See next page

Question 2   **(2 marks)**

Find the value of x, in terms of a and b, where the equation $y = 2ax^2 + b^2x$ has a derivative equal to zero given that a and b are positive constants.
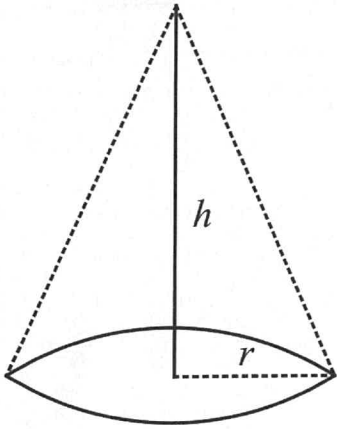
Question 3 (6 marks)

A tent in the shape of a cone is to be pitched. A bamboo frame is needed for the circumference of the base and the height of the cone. 8 metres of bamboo is to be used for the framework, represented by the solid lines in the diagram below.

(a) Show that the volume V, of the tent in terms of its radius r, is given by:

$$V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$

(2 marks)

(b) Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π. You **are not** required to prove it is a maximum. (4 marks)

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

2021
TEST 1

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work:　3 minutes
Working time for this section:　30 minutes
Marks available:　30 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 4   **(7 marks)**

An explosion produces a sound wave which expands through the air as a sphere. The radius increases at a rate of 300 m/s.

(a)   At what rate is the volume of the sphere increasing two seconds after the explosion?   **(3 marks)**

(b)   By using the Incremental formula, determine the percentage increase in the surface area of the sphere when the radius increases by two percent.   **(4 marks)**
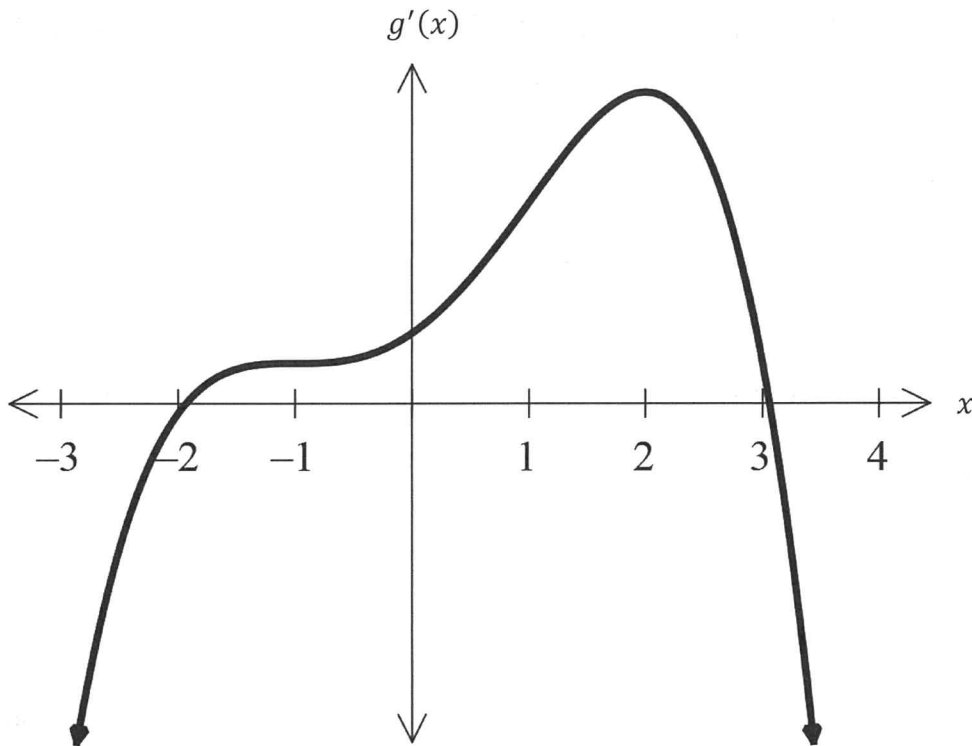
Question 5   (4 marks)

Consider the functions $f(x) = ax^3 + \dfrac{b}{x}$ with $f'(1) = 9$ and $f''(1) = 6$. Determine the values of a and b.

Question 6 **(4 marks)**

The graph of $y = g'(x)$ is sketched below.
On the same axes, sketch $y = g''(x)$.

Question 7   **(4 marks)**

A particle moves such that its displacement from the origin O, at time t seconds, is x metres, where:

$$x(t) = \frac{2t^4 - t^3 - 28t^2}{t+4} + 5 \qquad \text{for } t \geq 0.$$

Determine

(a)   the distance travelled by the particle in the first 5 seconds.   (2 marks)

(b)   the acceleration of the particle when $t = 5$.   (2 marks)

Question 8   **(3 marks)**

KTL Productions sells a product at a unit price of \$30. The cost of producing x items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a)   Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of x items.   (1 mark)

(b)   Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately \$10, given that more than 100 items were manufactured and sold.   (2 marks)
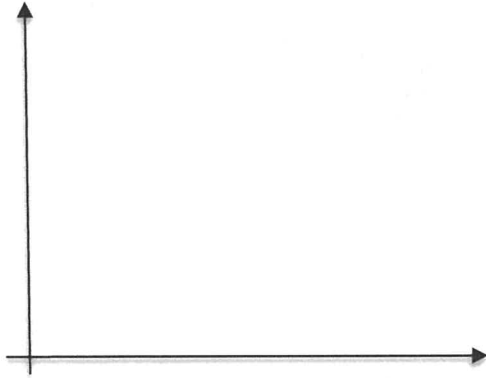
See next page

Question 9 (8 marks)

The population of a certain bacteria in a culture (in hundreds) is modelled by

$P = t + \sin(2t)$ for $2 \leq t \leq 6$, where t is time in weeks.

(a) Sketch the population on the axes below (2 marks)

(b) Find the exact value of t when the bacteria achieves a temporary peak in its population. State the population at this time. (4 marks)

(c) Find the maximum population of the bacteria in the interval $2 \leq t \leq 6$. (2 marks)

End of questions

Additional working space

Question number: _____

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____ –Solutions– _____

Teacher's name_____ MAR _____

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See next page

Question 1 (7 marks)

Differentiate with respect to x. Simplify your answers, leaving them in fully factorised form where appropriate.

(a) $\dfrac{2x+1}{8-3x}$ (2 marks)

$$\frac{u'v - uv'}{v^2} \Rightarrow \frac{2(8-3x) - (2x+1)(-3)}{(8-3x)^2} \checkmark \quad \left[\begin{matrix}\text{Quotient}\\\text{Rule}\end{matrix}\right]$$

$$= \frac{16 - 6x + 6x + 3}{(8-3x)^2}$$

$$= \boxed{\frac{19}{(8-3x)^2}} \checkmark \quad \left[\begin{matrix}\text{Fully}\\\text{Simplified}\end{matrix}\right]$$

(b) $(2x^4 + 2)(4 + x)^8$ (3 marks)

$u'v + uv'$

$$= 8x^3(4+x)^8 + (2x^4+2)\,8(4+x)^7 \times 1 \qquad \checkmark \quad [\text{Product Rule})$$

$$= 8(4+x)^7\left[x^3(4+x) + 2x^4 + 2\right] \qquad \checkmark \quad [\text{HCF}]$$

$$= 8(4+x)^7\left[4x^3 + x^4 + 2x^4 + 2\right]$$

$$= \boxed{8(4+x)^7(3x^4 + 4x^3 + 2)} \qquad \checkmark \quad [\text{Fully factorised}]$$

(c) $\sin(3x) - 4\cos 2x$ (2 marks)

$$= \underline{3\cos(3x)} + \underline{8\sin(2x)}$$
$$\quad\;\checkmark \qquad\qquad \checkmark \qquad [\text{Each part}]$$

See next page

Question 2 (2 marks)

Find the value of x, in terms of a and b, where the equation $y = 2ax^2 + b^2x$ has a derivative equal to zero given that a and b are positive constants.

If $y = 2ax^2 + b^2x$

$$\frac{dy}{dx} = 4ax + b^2$$

$$0 = 4ax + b^2 \qquad \checkmark \left[\frac{dy}{dx} = 0\right]$$

$$4ax = -b^2$$
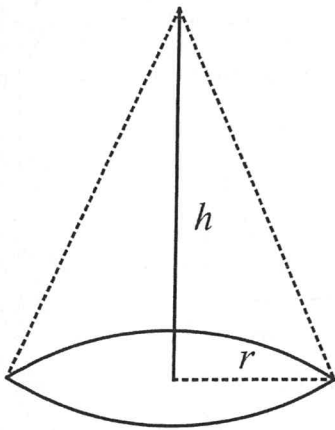
$$\boxed{x = \frac{-b^2}{4a}} \qquad \checkmark \left[x = \right]$$

2

Question 3   **(6 marks)**

A tent in the shape of a cone is to be pitched. A bamboo frame is needed for the circumference of the base and the height of the cone. 8 metres of bamboo is to be used for the framework, represented by the the solid lines in the diagram below.

(a)   Show that the volume V, of the tent in terms of its radius r, is given by:
$$V = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$
                                                                     (2 marks)

$$2\pi r + h = 8 \quad \Rightarrow \quad \underline{h = 8 - 2\pi r} \quad \checkmark$$

$$\quad [h \text{ subject}]$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2 (8 - 2\pi r) \quad \checkmark \quad [\text{sub in}]$$

$$\checkmark \quad = \frac{8}{3}\pi r^2 - \frac{2}{3}\pi^2 r^3$$

(b)   Determine the radius of the tent that will maximise the volume, leaving your answer in terms of π. You **are not** required to prove it is a maximum.   (4 marks)

$$\frac{dV}{dr} = \frac{16}{3}\pi r - 2\pi^2 r^2 \quad \Rightarrow \quad \frac{dV}{dr} = 0 \quad \checkmark \quad \left[\frac{dV}{dr}\right]$$

$$\therefore \quad \frac{16}{3}\pi r = 2\pi^2 r^2 \quad \checkmark \quad \left[\frac{dV}{dr} = 0, \text{ manipulates equation}\right]$$

$$\frac{16}{3} = 2\pi r$$

$$r = \frac{16}{6\pi} \quad \checkmark \quad [\text{isolates } r]$$

$$\boxed{r = \frac{8}{3\pi}\ m} \quad \checkmark \quad [\text{Answer with units}]$$

End of Questions

Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

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Reading time before commencing work: 3 minutes
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Marks available: 30 marks

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Question 4   **(7 marks)**

An explosion produces a sound wave which expands through the air as a sphere. The radius increases at a rate of 300 m/s.

(a)   At what rate is the volume of the sphere increasing two seconds after the explosion?   **(3 marks)**

$$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$$

We have $\frac{dr}{dt} = 300\,m/s$

$$= 4\pi r^2 \times 300 = \boxed{1200\pi r^2} \quad\checkmark \left[\begin{array}{c}\text{Chain}\\\text{Rule}\end{array}\right]$$

And $V = \frac{4\pi r^3}{3}$

At $t=2$, $r = 600$

$$\boxed{\therefore \frac{dv}{dr} = 4\pi r^2} \quad\checkmark \left[\frac{dv}{dr}\right]$$

$$\therefore \frac{dv}{dt} = 1200\pi (600)^2$$

$$= 432\ 000\ 000\,\pi \ m^3/sec \quad\checkmark \left[\text{Answ}\right]$$

OR $\qquad = \underline{1,357\ 168\ 026}\ m^3/sec$

(b)   By using the Incremental formula, determine the percentage increase in the surface area of the sphere when the radius increases by two percent.   **(4 marks)**

$$\delta A \approx \frac{dA}{dr} \times \delta r$$

$A_{sphere} = 4\pi r^2$

$$= 8\pi r \times 0.02 r$$

$$\boxed{\frac{dA}{dr} = 8\pi r} \quad\checkmark \left[\frac{dA}{dr}\right]$$

$$= \underline{0.16\pi r^2} \quad\checkmark \left[\text{Obtain } \delta A\right]$$

But $\boxed{\frac{\delta A}{A}} = \frac{0.16\pi r^2}{4\pi r^2} \quad\checkmark \left[\frac{\delta A}{A}\right]$

$$= \frac{0.16}{4} \implies \boxed{4\% \ increase} \quad\checkmark \left[\%\right]$$

See next page

Question 5   **(4 marks)**

Consider the functions $f(x) = ax^3 + \frac{b}{x}$ with $f'(1) = 9$ and $f''(1) = 6$. Determine the values of a and b.

$$f'(x) = 3ax^2 - bx^{-2}$$ ✓ $\left[\begin{array}{l}\text{Shows} \\ \text{1st derivative}\end{array}\right]$

$$\underline{9 = 3a - b} \quad —①$$ ✓ $[\text{forms equ}]$

$$f''(x) = 6ax + 2bx^{-3}$$

$$\underline{6 = 6a + 2b} \quad —②$$ ✓ $\left[\text{forms 2nd Equ}\right]$

Solve ① & ② Simultaneously

CPad

$$\boxed{\begin{array}{c} a = 2 \\ \hline b = -3 \end{array}}$$ ✓ $[\text{both correct}]$

4

Question 6

(4 marks)

The graph of $y = g'(x)$ is sketched below.
On the same axes, sketch $y = g''(x)$.

g '(x)

~ MAX

shape

$\left[x \text{ int } @ -1, 2 \right]$ ✓✓

$\left[\text{Approx Max} \right]$ ✓

$\left[\text{General Shape} \right]$ ✓
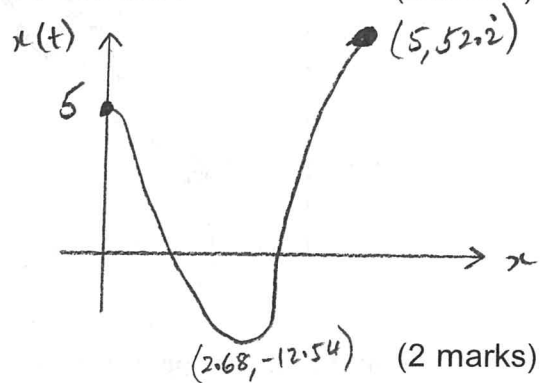
4

Question 7   **(4 marks)**

A particle moves such that its displacement from the origin 0, at time t seconds, is x metres, where:

$$x(t) = \frac{2t^4 - t^3 - 28t^2}{t+4} + 5 \qquad \text{for } t \geq 0.$$

Determine

(a) the distance travelled by the particle in the first 5 seconds.   **(2 marks)**

$Dist = (2 \times 12.54) + (52.2) + 5$ ✓

$= \boxed{82.3 m}$ ✓

$x(t)$

$(5, 52.2)$

5

$(2.68, -12.54)$

(b) the acceleration of the particle when $t = 5$.   **(2 marks)**

$\ddot{x}(5) = \underline{42.35\ ms^{-2}}$

on CPad ✓   ✓ [correct units]

Question 8   **(3 marks)**

KTL Productions sells a product at a unit price of \$30. The cost of producing x items is given by $C(x) = \frac{80x}{x+1} + 0.04x^2 + 500$.

(a) Find an expression for the profit $P(x)$ corresponding to the manufacture and sale of x items.   **(1 mark)**

$P(x) = 30x - \left(\frac{80x}{x+1} + 0.04x^2 + 500 \right)$ ✓

(b) Find how many items were manufactured and sold if the profit associated with the sale of the next item is approximately \$10, given that more than 100 items were manufactured and sold.   **(2 marks)**

✓ $\boxed{P'(x) = 10}$ $\Rightarrow$ $x = 1$ or 250

$\underline{250}$ items manufactured & sold (Accept 249)
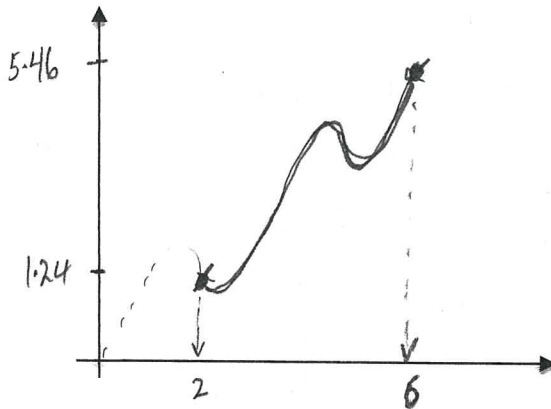✓

See next page

7

Question 9   **(8 marks)**

The population of a certain bacteria in a culture (in hundreds) is modelled by
$P = t + \sin(2t)$ for $2 \le t \le 6$, where t is time in weeks.

(a)   Sketch the population on the axes below   (2 marks)

✓ [General Shape]

✓ [Has end pts [2,6]]

(b)   Find the exact value of t when the bacteria achieve a temporary peak in its
population. State the population at this time.   (4 marks)

$\dfrac{dP}{dt} = 0 \quad \Rightarrow \quad 1 + 2\cos 2t = 0$

✓ $\left[\dfrac{dP}{dt} = 0\right]$ for $4 \le 2t \le 12$

$\cos 2t = -\frac{1}{2}$

$2t = \dfrac{8\pi}{3}, \ \dfrac{10\pi}{3}$ Represents min

MAX $\therefore \ t = \boxed{\dfrac{4\pi}{3}} \text{ OR } \boxed{\dfrac{5\pi}{3}}$ ✓ ✓ [t-value]

$\underset{4.188}{} \quad \underset{5.23}{}$

Population at this time $P = \dfrac{4\pi}{3} + \sin\left(\dfrac{8\pi}{3}\right)$ ✓ [sub in]

$P = 5.0548 \ (\times 100)$

$P = \underline{505}$ at this time ✓ [Population]

(b)   Find the maximum population of the bacteria in the interval $2 \le t \le 6$.   (2 marks)

$P\big|_{t=6} = \boxed{546}$ ✓✓ [only 1mk if 5.46]

End of questions

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